Curvature quintessence matched with observational data

S. Capozziello[*], V. F. Cardone[†], S. Carloni[‡], A. Troisi[§]

Dipartimento di Fisica"E. R. Caianiello",
Università di Salerno, I-84081 Baronissi, Salerno,
Istituto Nazionale di Fisica Nucleare, sez. di Napoli,
Gruppo Collegato di Salerno,
Via S. Allende-84081 Baronissi (SA), Italy

February 2, 2008

Abstract

Quintessence issues can be achieved by taking into account higher order curvature invariants into the effective action of gravitational field. Such an approach is naturally related to fundamental theories of quantum gravity which predict higher order terms in loop expansion of quantum fields in curved space-times. In this framework, we obtain a class of cosmological solutions which are fitted against cosmological data. We reproduce encouraging results able to fit high redshift supernovae and WMAP observations. The age of the universe and other cosmological parameters are discussed in this context.

PACS number(s): 98.80.Cq, 98.80. Hw, 04.20.Jb, 04.50

[*]capozziello@sa.infn.it
[†]winny@na.infn.it
[‡]carloni@sa.infn.it
[§]antro@sa.infn.it

1   Introduction

The existence of a dark energy term into cosmological dynamics has become a paradigm in the last five years since several observational campaigns give reliable indications on the apparent acceleration of the universe.

High redshift supernovae surveys [1, 2], CMBR data [3, 4, 5], Sunyaev-Zeldovich / X-ray methods [6] and other approaches provide a new picture of the universe. It can be represented as an isotropic, homogeneous, spatially flat 4-dim manifold filled with about 30% of baryonic and non-baryonic matter and about 70% of dark energy, simply referred as "cosmological component".

Also the very recent WMAP data [7] confirm such a picture with extremely low uncertainties in the estimate of cosmological parameters.

It is evident that the cosmological component should be the ingredient capable of generating the accelerated expansion, but, till now, its real nature is a puzzle which seems far to be solved.

Many approaches has been developed. Cosmological constant is the most straightforward candidate, however it is ruled out since its observational value (constrained by observations) differs of 120 order of magnitude from the theoretical prediction of QCD [8, 9, 10]. This argument does not allow to interpret cosmological constant as the vacuum energy of gravitational field unless an evolutionary mechanism is invoked in order to explain dynamics starting from the huge early values of energy (*cosmological constant problem*). Besides, the observed comparable amounts (in order of magnitude) of matter and dark energy sets a strong fine-tuning problem which cannot be overcome considering wide ranges of initial data (*coincidence problem*) [11].

A second approach is to consider the cosmological component as a dynamical term. This scheme, usually called *quintessence*, can be achieved adding scalar fields into Einstein gravity. Such scalar fields are assumed rolling their interaction potential [11, 12], as in the case of inflation but in a very different energetic regime (today instead of early universe). However major shortcomings come out due, essentially, to the *ad hoc* forms of self-interaction potential. Several forms of potential (inverse power law, exponential, etc, [9, 11]) achieve quintessence prescriptions (e.g. accelerated behaviour, $\Omega_\Lambda \simeq 0.7$, coincidence problem, etc.) but none of them seems to be directly related to some fundamental quantum field theory. The situation is, somehow, similar to that of inflationary cosmology: inflation is a good paradigm but no single model matches all requests.

On the other hand, alternative approaches can be pursued starting from some fundamental theory [13, 14, 15]. These schemes aim to improve the quintessence approach overcoming the problem of scalar field potential, generating a dynamical source for dark energy as an intrinsic feature. The goal would be to obtain a comprehensive model capable of linking the picture of early universe to the today observed one; that is, a model derived from some effective theory of quantum gravity which, through an inflationary period results in the today accelerated Friedmann expansion driven by some Ω_Λ-term. From this point of view, theories including torsion [16, 17, 18] or higher order curvature

invariants [19] naturally come into the game. In fact, the former ones allow to include spin matter fields at a fundamental level in General Relativity, while the latter ones come out in every quantization scheme of matter fields in curved space-time [9, 20, 21]. Specifically higher-order terms in curvature invariants as $R^2, R_{\mu\nu}R^{\mu\nu}, R\Box R$ are inescapable if we want to obtain an effective action of gravity closed to the Planck epoch. This scheme naturally give rise to inflationary behaviours [22, 23, 24].

In [19], it is investigated the possibility that such terms could act also today as a non-clustered form of dark-energy, providing accelerated behaviours. Besides, other authors have recently discussed curvature self-interactions of cosmic fluid to get cosmological constant [25].

In this paper, we want better to detail such an approach. We provide exact solutions and, following the scheme in [18], we try to obtain observational constraints on the so called *Curvature Quintessence* in order to see if curvature contributions can actually match the data of recent surveys.

The paper is organized as follows. In Sec.2, we summarize the curvature quintessence approach and derive a class of exact cosmological solutions. Sect.3 is devoted to the matching with observational data. In particular, we fit our solutions against the SNIa data and derive the age of the universe. A further discussion is carried out in Sec.4 considering the very recent WMAP data, which seem to better constrain the parameters of cosmological solutions. Sec.5 is devoted to the conclusions.

2   Curvature Quintessence

A generic fourth–order theory of gravity, in four dimensions, is given by the action [19],

$$\mathcal{A} = \int d^4x \sqrt{-g} \left[f(R) + \mathcal{L}_{(matter)} \right] , \tag{1}$$

where $f(R)$ is a function of Ricci scalar R and $\mathcal{L}_{(matter)}$ is the standard matter Lagrangian density. We are using physical units $8\pi G_N = c = \hbar = 1$. The field equations are

$$f'(R)R_{\alpha\beta} - \frac{1}{2}f(R)g_{\alpha\beta} = f'(R)^{;\mu\nu}(g_{\alpha\mu}g_{\beta\nu} - g_{\alpha\beta}g_{\mu\nu}) + \tilde{T}_{\alpha\beta}^{(matter)} , \tag{2}$$

which can be recast in the more expressive form

$$G_{\alpha\beta} = R_{\alpha\beta} - \frac{1}{2}g_{\alpha\beta}R = T_{\alpha\beta}^{(curv)} + T_{\alpha\beta}^{(matter)} , \tag{3}$$

where an stress-energy tensor has been defined for the curvature contributes

$$T_{\alpha\beta}^{(curv)} = \frac{1}{f'(R)} \left\{ \frac{1}{2}g_{\alpha\beta}[f(R) - Rf'(R)] + f'(R)^{;\mu\nu}(g_{\alpha\mu}g_{\beta\nu} - g_{\alpha\beta}g_{\mu\nu}) \right\} \tag{4}$$

2

and

$$T_{\alpha\beta}^{(matter)} = \frac{1}{f'(R)} \tilde{T}_{\alpha\beta}^{(matter)} , \tag{5}$$

is the stress-energy tensor of matter. We have taken into account the nontrivial coupling to geometry; prime means the derivative with respect to R. If $f(R) = R + 2\Lambda$, we recover the standard second–order gravity.

In a Friedmann-Robertson-Walker (FRW) metric, the action (1) reduces to the point-like one:

$$\mathcal{A}_{(curv)} = \int dt \left[\mathcal{L}(a, \dot{a}; R, \dot{R}) + \mathcal{L}_{(matter)} \right] \tag{6}$$

where the dot means the derivative with respect to the cosmic time. In this case the scale factor a and the Ricci scalar R are the canonical variables [26, 27].

It has to be stressed that the definition of R in terms of $a, \dot{a}, \ddot{a}$ introduces a constraint in the action (6) [19], by which we obtain by the Lagrange multiplier technique the lagrangian

$$\mathcal{L} = \mathcal{L}_{(curv)} + \mathcal{L}_{(matter)} = a^3 \left[f(R) - R f'(R) \right] + 6 a \dot{a}^2 f'(R) +$$

$$+ 6 a^2 \dot{a} \dot{R} f''(R) - 6 k a f'(R) + a^3 p_{(matter)} , \tag{7}$$

(the standard fluid matter contribution acts essentially as a pressure term [28]). The Euler-Lagrange equations coming from (7) give the second order system:

$$2 \left(\frac{\ddot{a}}{a} \right) + \left(\frac{\dot{a}}{a} \right)^2 + \frac{k}{a^2} = -p_{(tot)} , \tag{8}$$

and

$$f''(R) \left\{ R + 6 \left[\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a} \right)^2 + \frac{k}{a^2} \right] \right\} = 0 , \tag{9}$$

constrained by the energy condition

$$\left(\frac{\dot{a}}{a} \right)^2 + \frac{k}{a^2} = \frac{1}{3} \rho_{(tot)} . \tag{10}$$

Using Eq.(10), it is possible to write down Eq.(8) as

$$\left(\frac{\ddot{a}}{a} \right) = -\frac{1}{6} \left[\rho_{(tot)} + 3 p_{(tot)} \right] . \tag{11}$$

The accelerated or decelerated behaviour of the scale factor depends on the r.h.s. of (11). The accelerated behaviour is achieved if

$$\rho_{(tot)} + 3 p_{(tot)} < 0 . \tag{12}$$

To understand the actual effect of these terms, we can distinguish between the matter and the geometrical contributions

$$p_{(tot)} = p_{(curv)} + p_{(matter)} \qquad \rho_{(tot)} = \rho_{(curv)} + \rho_{(matter)} . \tag{13}$$

3

Assuming that all matter components have non-negative pressure, Eq.(12) becomes:

$$\rho_{(curv)} > \frac{1}{3}\rho_{(tot)} \, . \tag{14}$$

The curvature contributions can be specified by considering the stress-energy tensor (4); we obtain a curvature pressure

$$p_{(curv)} = \frac{1}{f'(R)} \left\{ 2 \left(\frac{\dot{a}}{a}\right) \dot{R}f''(R) + \ddot{R}f''(R) + \dot{R}^2 f'''(R) - \frac{1}{2}[f(R) - Rf'(R)] \right\} , \tag{15}$$

and a curvature energy-density:

$$\rho_{(curv)} = \frac{1}{f'(R)} \left\{ \frac{1}{2}[f(R) - Rf'(R)] - 3 \left(\frac{\dot{a}}{a}\right) \dot{R}f''(R) \right\} , \tag{16}$$

which account for the geometrical contributions into the thermodynamical variables.
It is clear that the form of $f(R)$ plays an essential role for this model. We choose the $f(R)$ function as a generic power law of the scalar curvature and we ask for power law solutions of the scale factor.
Summarizing, we consider:

$$f(R) = f_0 R^n \, , \qquad a(t) = a_0 \left(\frac{t}{t_0}\right)^\alpha \, . \tag{17}$$

The interesting cases are for $\alpha \geq 1$ which give rise to accelerated expansion.
Let us now concentrate now concentrate on the case with $\rho_{(matter)} = 0$. Inserting Eqs.(17) into the above dynamical system, for a spatially flat space-time ($k = 0$), we obtain an algebraic system for the parameters n and α

$$\begin{cases} \alpha[\alpha(n-2) + 2n^2 - 3n + 1] = 0 \\ \\ \alpha[n^2 + \alpha(n - 2 - n - 1)] = n(n-1)(2n-1) \end{cases} \tag{18}$$

from which the allowed solutions:

$$\alpha = 0 \; \rightarrow \; n = 0, \; 1/2, \; 1$$

$$\alpha = \frac{2n^2 - 3n + 1}{2 - n} \, , \; \forall n \; \text{but} \; n \neq 2 \, . \tag{19}$$

The solutions with $\alpha = 0$ are not interesting since they provide static cosmologies with a non evolving scale factor[1]. On the other hand, the cases with generic α and n furnish an entire family of significative cosmological models. By the plot in Fig.1 we see
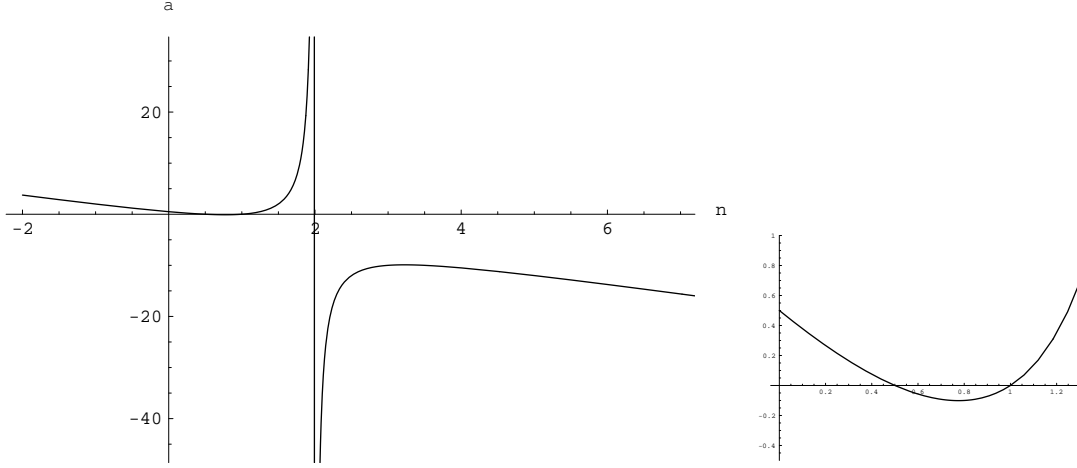
4

Figure 1: The behaviour of α in term of n. It is evident a region in which the power of the scale factor is more than one and a region in which it is always negative. The plot on the right represent the values of n between 0 and 1.5.

that such a family of solutions admit negative and positive values of α which give rise to accelerated behaviours.

Using Eqs.(15) and (16) we can also deduce the state equation (the barotropic index) for the family of solution $\alpha = \dfrac{2n^2 - 3n + 1}{2 - n}$. We have

$$w_{(curv)} = -\left(\frac{6n^2 - 7n - 1}{6n^2 - 9n + 3}\right)\,, \tag{20}$$

which clearly is $w_{(curv)} \to -1$ for $n \to \infty$. This fact shows that the approach is compatible with the recovering of a cosmological constant.

Eq.(20) is plotted in Fig.2. From the two plots, we can observe that the accelerated behaviour is allowed only for $w_{(curv)} < 0$ as requested for a cosmological fluid with negative pressure.
The whole approach seems intriguing. In fact we are able to describe the accelerated phase of universe expansion simply as an effect of higher order curvature terms which provide an effective negative pressure contribute. In order to see if such behaviour is possible for today epoch we have to match our model with observational data.

---

[1]This result match with the standard General Relativity case (n=1) in absence of matter.
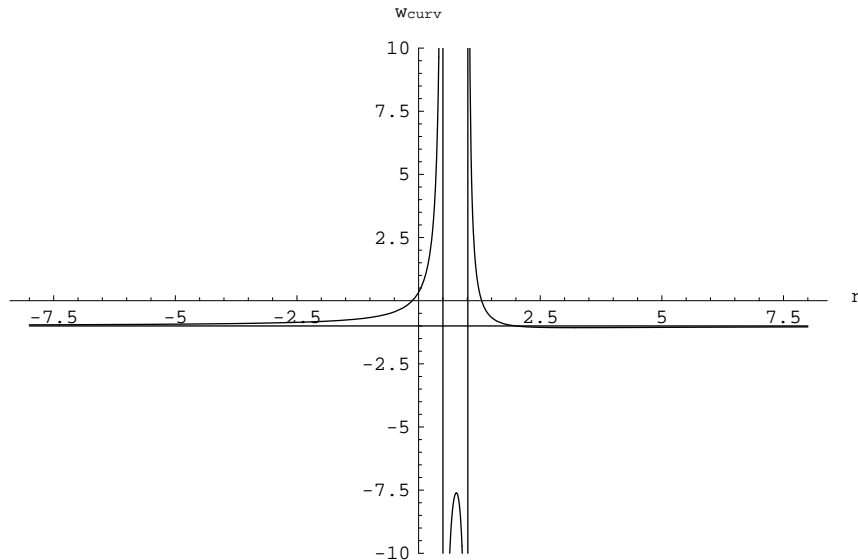
Figure 2: Behaviour of w_{curv} against n. We have drawn a line for the value $w_{curv} = -1$ emphasizing the cosmological constant state equation value.

3 Matching with SNIa observations

To verify if the curvature quintessence approach is an interesting perspective, we have to match the model with the observational data. In this way, we can constrain the parameters of the theory to significant values. First we compare our theoretical setting with the SNeIa results. As a further analysis, we check also the capability of our model in the universe age prediction.

We have stressed in Sec.1 that SNeIa observations have represented a cornerstone in the recent cosmology, pointing out that we live in an expanding accelerating universe. This result has been possible in relation to the feature of supernovae to be considered standard candles via the *Phillips amplitude-luminosity relation*.

To test our cosmological model, we consider supernovae observations reported in [1] [2]. We have compiled a combined sample of 79 SNeIa discarding 6 likely outliers SNeIa as discussed in [1].

Starting from these data, it is possible to perform a comparison between the theoretical expression of the distance modulus

$$\mu(z) = 5 \log \frac{c}{H_0} d_L(z) + 25 \,, \tag{21}$$

and its experimental value for SNeIa, z is the redshift (c is the light speed, hereafter we will use standard units). In general, the luminosity distance d_L can be expressed as:

6

$$d_z = \frac{c}{H_0}(1+z)\int_0^z \frac{1}{E(\zeta)}d\zeta\,, \tag{22}$$

where $E(\zeta) = \dfrac{H}{H_0}$. From (17), the Hubble parameter is

$$H(z) = H_0(1+z)^{1/\alpha}\,, \tag{23}$$

where α depends on n as in Eq.(19). A fit by SNeIa data provides significant values of the parameter n which assigns the curvature function.

The best fit is performed as in [29] minimizing the χ^2 calculated between the theoretical and the observational value of distance modulus:

$$\chi^2(H_0, n) = \sum_i \frac{[\mu_i^{theor}(z_i|H_0, n) - \mu_i^{obs}]^2}{\sigma_{\mu_0,i}^2 + \sigma_{mz,i}^2} \tag{24}$$

for the above considered data [1, 2].

Now, the luminosity distance becomes:

$$d_L(z, H_0, n) = \frac{c}{H_0}(1+z)\int_0^z (1+\zeta)^{-1/\alpha}d\zeta \tag{25}$$

which, after integration, gives

$$d_L(z, H_0, n) = \frac{c}{H_0}\left(\frac{\alpha}{\alpha-1}\right)(1+z)\left[(1+z)^{\frac{\alpha}{\alpha-1}} - 1\right]. \tag{26}$$

This expression is not defined for $\alpha = 0, 1$ which physically correspond to static universes and Milne ones; such models are not interesting for our purposes. The range of n can be divided into intervals taking into account the existence of singularities in (26). Thus, the fit is performed in five intervals of n, which are: $n < \frac{1}{2}(1-\sqrt{3})$, $\frac{1}{2}(1-\sqrt{3}) < n < \frac{1}{2}$, $\frac{1}{2} < n < 1$, $1 < n < \frac{1}{2}(1+\sqrt{3})$, $n > \frac{1}{2}(1+\sqrt{3})$.

In order to define a limit for H_0, we have to note that the Hubble parameter, as a function of n, has the same trend of α showed in Fig.1. We find that for n lower than –100, the trend is strictly increasing while for n positive, greater than 100, it is strictly decreasing. In relation to this feature, we have tested the values of n ranging in these limits because, as we shall see below, out of this range, the value of the age of universe becomes manifestly non-physically significant. The results of the fit are showed in Table 1. In Fig.3,

Range	$H_0^{best}(km\,s^{-1}Mpc^{-1})$	n^{best}	χ^2
$-100 < n < 1/2(1-\sqrt{3})$	65	-0.73	1.003
$1/2(1-\sqrt{3}) < n < 1/2$	63	-0.36	1.160
$1/2 < n < 1$	100	0.78	348.97
$1 < n < 1/2(1+\sqrt{3})$	62	1.36	1.182
$1/2(1+\sqrt{3}) < n < 3$	65	1.45	1.003
$3 < n < 100$	70	100	1.418

Table 1: Results obtained by fitting the curvature quintessence models against SNeIa data. First column indicates the range of n, column two gives the relative best fit value of H_0, column three n^{best}, column four the χ^2 index.

we present the different contour plots for each evaluated range.

From Tab.1 and Fig.3, it is evident that only in the ranges $1/2(1-\sqrt{3}) < n < 1/2$ and $1/2 < n < 1$ we can achieve a constraint on the values of n. Besides, we can exclude the range $1/2 < n < 1$ as physically not interesting, in relation to the best fit value of H_0 and the χ^2 result. In the other cases, the test gives significant best-fit values both for Hubble parameter and χ^2. But the contour plots are completely degenerate with respect to the n parameter. This occurrence suggests that χ^2 varies slightly with respect to n in this cases, hindering the possibility to constrain n.
In order to discriminate the significant value of n, we have also performed a test of the model in relation to the capability of estimating the age of the universe.

The age of the universe can be simply obtained, from a theoretical point of view, if one knows the value of the Hubble parameter. In our case, from the definition of H, and using the relations (17), we have:

$$t = \alpha H^{-1} \tag{27}$$

which is

$$t = \left(\frac{2n^2 - 3n + 1}{2 - n}\right) H^{-1}. \tag{28}$$

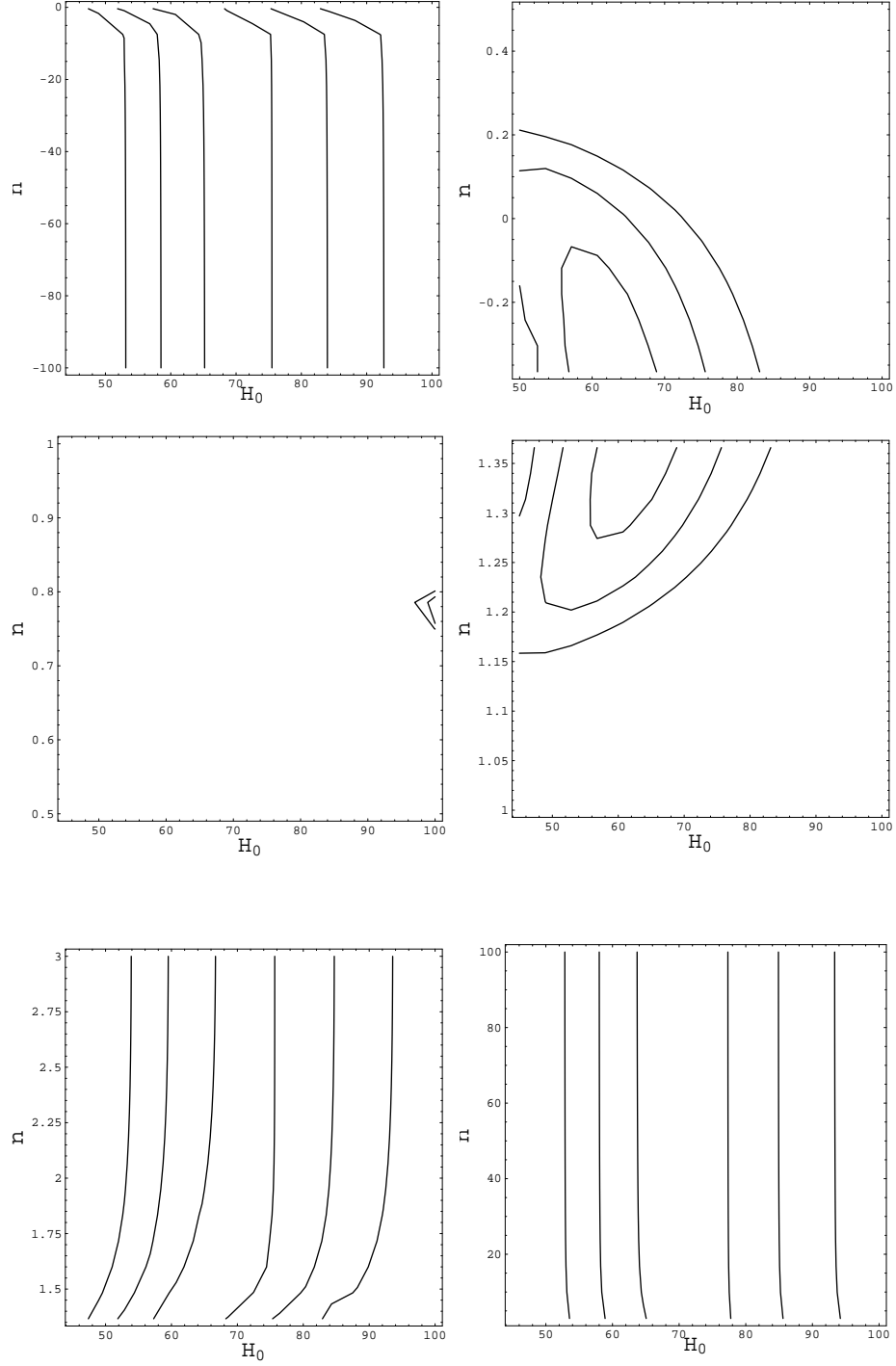The age of universe can be obtained substituting $H = H_0$ in (28).

8

Figure 3: Contour plots for the considered n-ranges using the SNIa data. It is evident that in the interval $1/2 < n < 1$ the models are non physical. The ranges $1/2(1 - \sqrt{3}) < n < 1/2$ and $1/2 < n < 1$ give some indications on the best fit value of n, in the other cases the matching with SNIa is completely degenerate.

9

Range	$\Delta H(km\ s^{-1}Mpc^{-1})$	Δn	$t(n^{best})(Gyr)$
$-100 < n < 1/2(1-\sqrt{3})$	$50-80$	$-0.67 \leq n < -0.37$	23.4
$1/2(1-\sqrt{3}) < n < 1/2$	$57-69$	$-0.37 < n \leq -0.07$	15.6
$1 < n < 1/2(1+\sqrt{3})$	$56-70$	$1.28 \leq n < 1.36$	15.3
$1/2(1+\sqrt{3}) < n < 2$	$54-78$	$1.37 < n \leq 1.43$	24.6

Table 2: The results of the age test. In the first column is presented the tested range. Second column shows the 3σ-range for H_0 obtained by supernovae test, while in the third we give the n intervals, i.e. the values of n which allow to obtain ages of the universe ranging between $10Gyr$ and 18 Gyr. In the last column, the best fit age values of each interval are reported.

We evaluate the age taking into account the intervals of n and the 3σ-range of variability of the Hubble parameter deduced by the Supernovae fit. We have considered, as good predictions, age estimates included between $10Gyr$ and $18Gyr$.

By this test, we are able of refine the allowed values of n. The results are shown in Table 2. First of all, we discard the intervals of n which give negative values of t. Eq. (27) shows that negative values of the age of the universe are obtained for negative values of α, so we have to exclude the range $1/2 < n < 1$ and $n > 2$ (Fig.1). Conversely, the other ranges, tested by SNIa fit (Tab.1), become narrower, strongly constraining n.

A further check for the allowed values of n is to verify if the interesting ranges of n provide also accelerated expansion rates. This test can be easily performed considering the definition of the deceleration parameter $q_0 = -\left.\left(\frac{\ddot{a}a}{\dot{a}^2}\right)\right|_0$, using the relation (17) and the definition of α in term of n. To obtain an accelerated expanding behaviour, the scale factor $a(t) = a_0 t^\alpha$ has to get negative or positive values of α greater than one. We obtain is that only the intervals $-0.67 \leq n \leq 0.37$ and $1.37 \leq n \leq 1.43$ provide a negative deceleration parameter with $\alpha > 1$. Conversely the other two intervals of Tab.2 do not give interesting cosmological dynamics, being $q_0 > 0$ and $0 < \alpha < 1$.

4 WMAP Age test

A further test of the model can be performed by the age estimate obtained by the WMAP campaign. The WMAP (*Wilkinson Microwave Anisotropy Probe*) mission aims to determine the geometry, the content and the evolution of the universe through a full sky map of the temperature anisotropies of cosmic microwave background radiation [7]. The

first year observational results has been published since few months. These observations indicate as the best fit model a Λ-term cosmological model with about 70%-content of cosmological origin, Hubble constant value $71^{+0.04}_{-0.03}\,km\,s^{-1}Mpc^{-1}$ and an age estimate of $13.7^{+0.2}_{-0.2}Gyr$.

Using this last result, we can improve the constraints on n in relation to the very low error (1%) of WMAP age estimator.

We use the same approach of the previous section, the only difference is to consider as physically interesting only the age prediction ranging between $13.5Gyr$ and $13.9Gyr$. The results are shown in Tab.3.

It is evident that this test narrows the range of physical interest. If we take into account also the capability of providing acceleration, we obtain that the interesting values of n are $-0.450 \leq n < -0.370$ and $1.366 < n \leq 1.376$.

These results could represent a selection for the allowed form of fourth-order gravity action as a power law of curvature Ricci scalar.

$Range$	$\Delta H(km\,s^{-1}Mpc^{-1})$	Δn	q_0
$-100 < n < 1/2(1-\sqrt{3})$	$50-80$	$-0.450 \leq n < -0.370$	< 0
$1/2(1-\sqrt{3}) < n < 1/2$	$57-69$	$-0.345 < n \leq -0.225$	> 0
$1 < n < 1/2(1+\sqrt{3})$	$56-70$	$1.330 \leq n < 1.360$	> 0
$1/2(1+\sqrt{3}) < n < 2$	$54-78$	$1.366 < n \leq 1.376$	< 0

Table 3: The results of the age test for the curvature quintessence model based on the prediction of WMAP observations. In the last column we show the sign of the deceleration parameter as a testify of the potential accelerating rate of the model.

5   Conclusions

In this paper we have analyzed a geometrical approach to quintessence given by considering a fourth-order theory of gravity and we have matched the cosmological models derived with observational data. This scheme, proposed as *curvature quintessence* [19], represents an approach to describe an accelerated expanding universe dominated by a cosmological component without using scalar fields.

We stress that such an approach has a natural background in several attempts of quantize gravity, because higher-order curvature invariants are generated to renormalize quantum field theories on curved space times [21].

We have studied curvature quintessence neglecting matter as a first significative approximation. This approximation is possible if we consider that, at one loop level, matter and gravity have the same "weight" in early universe [20] and today the amount of cosmological and matter components, at cosmological density level, are comparable. In both

regimes, the overall evolution can be assigned only by the cosmological component which gives a good approximation of dynamics. We ask for a power law functions for the action (in terms of Ricci curvature scalar) and for the scale factor (in terms of cosmological time). With these choices, we obtain a family of exact solutions so that the model is characterized by one parameter (specifically n). To check these solutions, we have fitted the model with observational data. A straightforward test is a comparison with SNIa observations referring to the well known data of SCP (*Supernovae Cosmology Project* [1]) and of HZT (*High-Z search Team* [2]).

The model fits these data and provides a constrain on the parameter n. Unfortunately this test is not decisive to feature curvature quintessence since the likelihood curves are degenerate with respect to n.

To improve this result, we have performed a test with the age of the universe.

In a first case, our test has been conceived considering, as good estimates for age of universe, values ranging between $10 Gyr$ and $18\ Gyr$. Considering that we take into account only accelerated dynamics, we deduced that for $-0.67 \leq n \leq 0.37$ and $1.37 \leq n \leq 1.43$ curvature quintessence is a model capable of mimicking the actual universe.

In order to better refine these ranges, we have then considered a test based on WMAP age evaluation.

In this case, the age ranges between $13.5 Gyr$ and $13.9 Gyr$. This fact reduces the allowed intervals of the parameter n, which are $-0.450 \leq n < -0.370$ and $1.366 < n < 1.376$. In conclusion, we can say that a fourth order theory of gravity of the form

$$f(R) = f_0 R^{1+\varepsilon} \tag{29}$$

with $\varepsilon \simeq -0.6$ or $\varepsilon \simeq 0.4$ can give rise to reliable cosmological models which well fit SNeIa and WMAP data.

In this sense, we need only "small" corrections to second order Einstein gravity action in order to achieve quintessence issues.

Indications in this sense can be found also in a detailed analysis of $f(R)$ cosmological models performed against CMBR constraints, as shown in [30].

References

[1] S. Perlmutter et al. *Ap. J.* **483**, 565 (1997).
    S. Perlmutter et al. *Nature* **391**, 51 (1998).
    S. Perlmutter et al. *Ap. J.* **517**, 565 (1999).

[2] B.P. Schmidt et al. *Ap. J.* **507**, 46 (1998).
    A.G. Riess et al. *Ap. J.* **116**, 1009 (1998).

[3] P. de Bernardis et al. *Nature* **404**, 955 (2000).

[4] A. Balbi et al.*Ap. J.* **558**, L145-L146 (2001); R. Stompor et al. *Ap. J.* **561**, L7-L10 (2001).

[5] G.F. Smoot, SLAC Beam Line **23N3** 2, (1993); C. L. Bennet et al. *Ap. J.* **464**, L1 (1996); A. H. Jaffe et al *Phys. Rev. Lett.* **86**, 3475 (2000).

[6] Y. Repaheli, astro-ph/0211422.

[7] C.L. Bennet et al. (WMAP collaboration) astro-ph/0302207 (2003).
D.N. Spergel et al.(WMAP collaboration) astro-ph/0302209 (2003).

[8] S. M. Carroll, Living Rev.Rel. **4**, 1 (2001).

[9] A. A. Starobinsky, V. Sahni, *Int. Journ. Mod. Phys.* **D9**, 373-444(2000).

[10] N. Straumann, astro-ph/0203330.

[11] R.R. Caldwell, R. Dave, P.J. Steinhardt, *Phys. Rev. Lett.* **80**, 1582 (1998).

[12] R. de Ritis et al., *Phys. Rev.* **D 62** (2000) 043506.
C. Rubano and J.D. Barrow, *Phys. Rev.* **D64** (2001) 127301.
C. Rubano and P. Scudellaro, *Gen. Relativ. Grav.* **34**, 1931 (2002).

[13] A. Ujjaini, V. Sahni, astro-ph/0203443; V. Sahni, Y. Shtanov, astro-ph/0202346.

[14] K. Freese, M. Lewis, astro-ph/0201229; Y. Wang, K. Freese, P. Gondolo, M. Lewis, astro-ph/0302064.

[15] N. Bilic, G. B. Tupper, R. D. Viollier, *Phys. Lett.* **B535** 17, (2002); J. C. Fabris, S. V. B. Goncalves, P. E. de Souza, astro-ph/0207430; A. Dev, J. S. Alcanitz, D. Jain, *Phys. Rev.* **D67**, 023515 (2003).

[16] S. Capozziello, G. Lambiase and C. Stornaiolo, *Ann. Phys.* **10**, 713 (2001).

[17] S. Capozziello, *Mod. Phys. Lett.* **A17**, 1621 (2002).

[18] S. Capozziello, V.F. Cardone, E. Piedipalumbo, M. Sereno, A. Troisi, *Int. Journ. Mod. Phys.* in printing, astro-ph/0209610.

[19] S. Capozziello, *Int. Journ. Mod. Phys.* **11**, 483 (2002).

[20] N.D. Birrell and P.C.W. Davies *Quantum Fields in Curved Space* (1982) Cambridge Univ. Press (Cambridge).

[21] I.L. Buchbinder, S.D. Odintsov, I.L. Shapiro, *Effective Action in Quantum Gravity* IOP Publishing (1992) Bristol.

[22] A.A. Starobinsky, *Phys. Lett.* **B 91** (1980) 99.

[23] R. Kerner, *Gen. Relativ. Grav.* **14**, 453, (1982).

[24] J. P. Duruisseau, R. Kerner, *Class. Quantum Grav.* **3**, 817, (1986).

[25] A. B. Balakin, D. Pavon, D. J. Schwartz, W. Zimdahl, astro-ph/0302150.

[26] S. Capozziello, R. de Ritis, and A.A. Marino *Gen. Rel. Grav.* **30** (1998) 1247.

[27] S. Capozziello, G. Lambiase, *Gen. Relativ. Grav.* **32**, 295 (2000).

[28] S. Capozziello, R. de Ritis, C. Rubano, P. Scudellaro, *Int. Journ. Mod. Phys.* **D 4** (1995) 767.

[29] Y. Wang, *Ap. J.* **536**, 531 (2000).

[30] J. Hwang, H. Noh, *Phys. Lett.* **506 B** (2001) 13.